U. S. SANITARY COMMISSION.

STATISTICAL BUREAU.

AGES

OF

U. S. VOLUNTEER SOLDIERY.

NEW YORK.
1866.

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CAMBRIDGE, January, 1866.

DEAR SIR, —

Inclosed is my Report on the Ages of Volunteers in the late war. The general results were communicated, by permission, to the National Academy of Sciences at its session in August last, and this Report was read before the Commission in November; but the Appendix, concerning the Ages of a Population in general, has been prepared since that time.

For most of the troops comprised within the limits of the present discussion, no descriptive muster-rolls exist. The Ages, Nativities, and Statures of those troops whose descriptions are on record will soon be made the subject of another Report.

I am, dear Sir,

Very respectfully and sincerely yours,

B. A. GOULD,
Actuary U. S. Sanitary Commission.

JOHN S. BLATCHFORD, ESQ.,
General Secretary U. S. Sanitary Commission.

CORRIGENDA.

Page	6,	line	23,	*for* face	*read* force.
"	10,	"	19,	" 28.4843	" 28.4850.
"	30,	"	28,	" Column 9	" Column 3.
"	39,	"	24,	" one million	" one hundred thousand.

AGES

OF

UNITED STATES VOLUNTEER SOLDIERY.

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1. *Introductory.*

On taking charge of the Statistical Department of the United States Sanitary Commission, in August, 1864, it was found that considerable progress had been made in collecting the ages of the soldiers of our volunteer regiments, — an investigation which had been suggested and commenced by Mr. Elliott, the accomplished and skilful statistician, who, not very long before, had relinquished the direction of this Bureau of the Commission.

Although the best use to be made of the materials appeared somewhat uncertain, it did not seem proper to discontinue inquiries already so far advanced; and the large experience of Mr. Elliott in matters connected with vital statistics gave assurance that valuable as well as interesting results were likely to be deduced from a thorough study of these data.

The collection of these materials was therefore continued and completed, by means of the muster-rolls on file at the War Department in Washington, to which access was courteously afforded by General E. D. Townsend, Acting Adjutant-General, and Colonel Samuel Breck, who was in charge of the rolls. Tables have thus been formed for twenty-seven States, Territories, or geographical groups, exhibiting the number of men at each year of age in the volunteer organizations, at the time of their muster into the service of the United States. The officers are tabulated as a distinct class; and the three arms of the military service — infantry, cavalry, and artillery — have been treated separately.

The original collection of the materials was principally made by Mr. T. J. O'CONNELL, until lately the efficient and accurate chief clerk of the Statistical Department, and was completed by Mr. E. A. WILSON. The tabulation has been made by Messrs. O'CONNELL, WILSON, A. A. BROOKE, and JOHN N. STOCKWELL; and the greater part of the computations has been performed by Mr. STOCKWELL alone, with great care, perseverance, and ability.

The recruits who joined these original regiments after their first organization and acceptance into the national service are not included; and the limits of the investigation have excluded all drafted men, substitutes, &c. Moreover, many regiments belonging within these limits are omitted, because organized since the collection of the data for the States to which they belong; but the number of these is comparatively small, and inadequate to exert any sensible effect upon the results. The degree of completeness may be seen by the following table, which shows the number and date of the latest regiment included in the collection.

Arkansas	2d	Infantry,	latest.	Mississippi	Marine Brig.		only organ'n.
California	4th	"	1862, Feb.	Missouri	34th	Infantry	1862, Dec.
Connecticut	28th	"	1862, Nov.	Nevada	1st	"	1864, June.
Delaware	2d	"	1861, Dec.	N. Hampshire	18th	"	1864, Sept.
Illinois	131st	"	1864, June.	New Jersey	25th	"	1862, Sept.
Indiana	115th	"	1863, Aug.	N. Mexico	4th	"	1863.
Iowa	48th	"	1865.	New York	177th	"	1863, June.
Kansas	15th	"	1863.	Ohio	128th	"	1863, Aug.
Kentucky	52d	"	1864.	Pennsylvania	155th	"	1863, Jan.
Louisiana	N. O.	Vols.	1864, May.	Rhode Island	12th	"	1862, Oct.
Maine	28th	Infantry	1864.	Tennessee	8th	"	1864.
Maryland	10th	"	1864, June.	Vermont	16th	"	1862, Oct.
Mass.	59th	"	1864, July.	W. Virginia	15th	"	1862, Sept.
Michigan	27th	"	1864, Aug.	Wash. Terr.	1st	"	only reg't.
Minnesota	10th	"	1864, Aug.	Wisconsin	53d	"	1864.

The total number of volunteers whose ages have thus been investigated is 1 049 457, of whom 1 012 273 were enlisted men, and 37 184 were commissioned officers. All except $1\frac{1}{2}$ per centum (.01495) of the men, and $3\frac{1}{3}$ per centum (.0331) of the officers, were between the ages of 18 and 46 years at the date of their enlistment or commission. Those beyond these limits have not been included in the determination of the general formulas,

so that these depend upon the statistics of ages for 1 032 600 men, of whom 35 953 were commissioned officers.*

The results have proved amenable to law in a higher degree than I had ventured to anticipate. Residual discordances exist, of course, between the numbers for each year of age, as derived from the tabulated records, and those indicated by the general formulas deduced from the whole series; yet where these discordances attain any essential magnitude, they may almost invariably be made to yield instructive and useful information.

The results attained, for that portion of the population who thus rushed to the field at their country's call, naturally suggest analogous inquiries regarding the white male population of the United States, and especially relative to the population of that portion of the country which furnished the volunteers under consideration. And it was not until after many unavailing efforts to obtain information as to the distribution of our population by ages, that the great deficiency of our knowledge of the facts and laws relative to this very important subject became manifest.

The only published attempt, of which I am aware, to classify the population of the United States according to years of age is very crude, and the method pursued yields results quite at variance from the truth. The only trustworthy facts are contained in the summaries of the census-returns; and the groups into which the population is there divided are altogether too large to permit the desired laws to be deduced with ease. It is earnestly to be hoped that in future census-publications the groups may be so made as to include intervals of age not greater than five years.

It thus became important, if only for the sake of comparison between the ages of the volunteer troops and that of the population whence they sprung, to subject the census of 1860 to a similar discussion. And I cannot but think that the results elicited might be advantageously employed, so far as they apply and extend, for the life-tables of our insurance and annuity offices. The life-curve for our American population is clearly diverse from the curve on which the present English tables are based;

* The prescribed limits of military age at the commencement of the rebellion were 18 and 45 years; but the large proportional number at the age of 45 seems to indicate that the law was so interpreted as to permit the acceptance of volunteers whose age at their last birthday did not exceed 45 years.

and it is a source of regret that the proper limits of the present investigation forbid its extension into the tempting fields of inquiry which their comparison suggests.

The fact which first attracts attention among the results of this research is the marked diversity between the distribution of the ages of officers and that of the enlisted men. Each follows a clearly manifest law; in each case the law is deducible with close approximation to the truth; so also is the law governing the ages of our population; yet each of the three is utterly different from the other two. The sources of the diversity may well be made the object of careful research, and not without a reasonable probability of useful results. Certain discordances between the recorded and the computed numbers for a few particular ages will be considered hereafter.

2. *Ages of the Enlisted Men.*

The grand total of the rank and file of the volunteers whose ages are included in this discussion is shown in the following tabular view, which exhibits the recorded age at last birthday for the entire number; although, as already stated, those under 18 or over 45 (last birthday), 15 626 in all, have been excluded from the general discussion. These excluded cases represent two classes, viz. the boys, chiefly drummers, musicians, &c., and the men who, although past the legal age, were so sturdy or earnest that the enrolling officers did not, at that time of great national peril, feel justified in insisting on an absolute compliance with the legal qualifications.

In the column entitled "Miscellaneous" are included all those organizations which do not belong strictly within the three principal arms of the military service, such as Engineers, Sharpshooters, Mounted Infantry, Coast Guards, Marine Brigades, &c., together with a few regiments or battalions for which the statistics were received after the special computations for Infantry, Cavalry, and Artillery had been completed, so that their incorporation with these would have required a repetition of the calculations without producing any essential change in the result.

TABLE I.

Classified Summary of Enlisted Volunteers.

Age at last birthday.	ACTUAL NUMBER OF MEN.				Total at each year of age.
	Infantry.	Cavalry.	Artillery.	Miscellaneous.	
13	113	5	0	9	127
14	288	15	2	25	330
15	636	49	21	67	773
16	2053	232	61	412	2758
17	4653	638	226	908	6425
18	103420	15013	5400	9642	133475
19	71226	9767	3439	5783	90215
20	56238	7864	2627	4329	71058
21	75978	12081	4416	4661	97136
22	57485	9096	3107	3703	73391
23	48954	7806	2759	3198	62717
24	40852	6361	2163	2719	52095
25	36383	5724	2012	2507	46626
26	31292	4831	1768	2352	40243
27	26369	4192	1505	2220	34286
28	27196	4318	1525	2273	35312
29	18833	2845	1087	1748	24513
30	21937	3251	1213	1959	28360
31	12814	2053	796	2301	17954
32	17038	2450	931	1548	21967
33	13678	1950	753	1598	17979
34	12004	1679	724	1333	15740
35	14558	2130	836	1456	18980
36	10437	1541	702	1377	14057
37	8782	1268	477	1293	11820
38	10025	1416	579	1326	13346
39	7200	979	416	1001	9596
40	10886	1441	649	1019	13995
41	5634	822	320	659	7435
42	8369	1199	535	826	10929
43	7900	1079	533	828	10340
44	12274	1851	796	1149	16070
45	5509	954	289	260	7012
46	737	105	45	80	967
47	541	74	34	63	712
48	532	73	31	63	699
49	354	60	17	38	469
50 & over.	1942	203	68	153	2366

The relative excess of the numbers at certain particular ages, and the corresponding defect at others, strikes the attention at the first glance. To the former class belong the ages, 21 years, most years divisible by 5 (excepting 20 and 45), and those divisible by 2; to the latter class belong most of those years of age whose last digit is 1 or 9. By determining the general law of distribution, we may obtain the measure of this excess, and thus throw light upon the origin of these discordances.

The following facts are also manifest, or readily deducible: —

Of the whole number, 1 012 273, about 1 per centum (.0102), were below, and a little more than one half as many (.0052) were above, the limits of military age, interpreted as between the ages 18 and 46.

Of the number 996 647, within these limits, —

	y.
The average age at last birthday is	25.3250
The average age at time of enlistment is . . .	25.8083
The age above and below which the numbers are equal is	23.477
There were of the age 18 years	13.27 per cent.
under 21 years	29.52 "
under 25 years	58.34 "
under 30 years	76.57 "

The very close accordance of the proportional numbers for the total face of about a million of men from all the loyal States, with those deduced * by Mr. ELLIOTT for less than 51 000 men from the single State of Massachusetts, is very striking. Tables for the individual States and groups of States, herewith presented, unite in corroborating the inference that this distribution is due to no special local influences, but to a general and overruling law, which varies but slightly through widely distant regions of our country, and seems scarcely affected by any influences dependent upon immigration from abroad.

This law, which was found by Mr. ELLIOTT to hold good also for the Massachusetts troops, shows the number of volunteers (enlisted men, not including officers) at each successive year of age to form a series of which the first differences are in geometrical progression.

When the ratio of this geometrical progression is unity, the

* "*On the Military Statistics of the United States of America,*" Proceedings of the International Statistical Congress,. V Session, 1863, p. 32.

progression becomes arithmetical; when, as in the present case, it is less than unity, we have a decreasing rate of change.

Let this ratio be denoted by h, and the number of men at any given year of age be

$$x_n = b + c\ (1 - h)\ h^n \tag{1}$$

so that the total number at and over that age will be

$$s_n = a - b\,n + c\,h^n \tag{2}$$

in which n denotes the excess of the age above 16 years, at which epoch

$$s_0 = a + c.$$

The constants a, b, c, h are to be determined, and we have

$$\Delta x_0 = c\ (1 - h)^2, \qquad \Delta x_n = c\,h^n\ (1 - h)^2, \qquad \Delta_m x_{mn} = c\,h^{mn}\ (1 - h^m)^2$$

whence

$$h^n = \frac{\Delta_m x_{mn}}{\Delta_m x_{(m+1)\,n}} \tag{3}$$

which enables us to determine h from the most convenient equidistant portions of the series.

The variation of the fundamental equation (2) gives for any change in the values of the constants

$$\partial s_n = \partial a - n\,\partial b + h^n\,\partial c + n\,c\,h^{n-1}\,\partial h, \tag{4}$$

by means of which, after an approximate value of h has been deduced from (3), and corresponding values of a, b, c derived from the numerical data for any four years, the corrected values of all four constants may be derived by the method of least squares.

The total number up to any given age, or the definite sum from x_0 to x_n, is evidently

$$s_0 - s_n = b\,n + c\ (1 - h^n) = \Sigma_0^n x \tag{5}$$

so that

$$-n + \frac{c}{b}\,h^n = \frac{1}{b}\ (c - \Sigma_0^n x)$$

or by (2)

$$= \frac{1}{b}\ (s_n - a). \tag{6}$$

Since the numerical values deduced from the tables belong not to the age n years, but to that age which corresponds to the average

for all the individuals between n and $n+1$ years, the constants deduced hold good also for the series of these mean ages; the successive annual arguments being really at intervals differing slightly from one year.

The age t corresponding to this average may be deduced for any year with sufficient accuracy for all practical purposes, by putting $n = t$ in the first member of equation (6), and using in the last member the value of $s_{n+\frac{1}{2}}$ instead of s_n, which gives

$$-t + \frac{c}{b} h^t = \frac{1}{b} (s_{n+\frac{1}{2}} - a). \qquad (7)$$

Similarly we may find the age corresponding to the average for any period of years. For this purpose we replace $s_{n+\frac{1}{2}}$ in the last member of the equation (7) by

$$\tfrac{1}{2} (s_n + s_{n'}) = a - \tfrac{1}{2} b (n + n') + \tfrac{1}{2} c (h^n + h^{n'})$$

and the corresponding value of t is the age equivalent to the average of the period included between n and n'.

Proceeding as above described, and, after the first approximate determination of h, a, b, c, from four conveniently situated and equidistant observed values of s_n, obtaining improved values for all four constants by the method of least squares, the formulas derived from the grand total of all the enlisted men of military age as presented in Table I. are these, which express the relative numbers for every ten thousand: —

$$\begin{aligned} x_n &= \quad + 77.04 \quad + 1156.0 \cdot 0.85362^n \\ s_n &= 2102.8 - 77.04\, n + 7897.2 \cdot 0.85362^n. \end{aligned}$$

With these values the fourth and seventh columns of Table II. are computed, the third and sixth columns showing the "observed," or recorded numbers, reduced to the same scale; and the fifth and eighth columns exhibiting the discordances between the calculated and observed values.

These discordances, although in one sense regular, inasmuch as the larger ones are apparently not the result of so-called accident, or, in other words, of the use of numbers insufficient to eliminate discordances of no palpable significance, are in another sense markedly devoid of regularity, inasmuch as the positive and negative signs alternate freely, and no decided indication seems to exist of a systematic deviation of the general formula.

TABLE II.

Grand Total of Enlisted Men.

Age at last birthday.	Number.	Proportion at and over given age.		Difference. (C. — O.)	Proportion at given age.		Difference. (C. — O.)
		Observed.	Calculated.		Observed.	Calculated.	
13	127						
14	330						
15	773						
16	2758						
17	6425						
18	133475	10000	10000	0	1339	1233	−106
19	90215	8661	8767	+106	905	1064	+159
20	71058	7756	7703	− 53	713	919	+206
21	97136	7043	6784	−259	975	796	−179
22	73391	6068	5988	− 80	736	691	− 45
23	62717	5332	5297	− 35	629	601	− 28
24	52095	4703	4696	− 7	523	524	+ 1
25	46626	4180	4172	− 8	468	460	− 8
26	40243	3712	3712	0	404	403	− 1
27	34286	3308	3309	+ 1	344	355	+ 11
28	35312	2964	2954	− 10	354	315	− 39
29	24513	2610	2641	+ 31	246	280	+ 34
30	28360	2364	2361	− 3	285	250	− 35
31	17954	2079	2111	+ 32	181	225	+ 44
32	21967	1898	1886	− 12	221	203	− 18
33	17979	1677	1683	+ 6	181	185	+ 4
34	15740	1496	1498	+ 2	158	169	+ 11
35	18980	1338	1329	− 9	191	156	− 35
36	14057	1147	1173	+ 26	141	144	+ 3
37	11820	1006	1029	+ 23	118	134	+ 16
38	13346	888	895	+ 7	133	126	− 7
39	9596	755	769	+ 14	96	118	+ 22
40	13995	659	651	− 8	141	112	− 29
41	7435	518	539	+ 21	74	107	+ 33
42	10929	444	432	− 12	109	103	− 6
43	10340	335	329	− 6	104	99	− 5
44	16070	231	230	− 1	161	96	− 65
45	7012	70	134	+ 64	70	93	+ 23
46	967						
47	712						
48	699						
49	469						
50 & over.	2366						

The trustworthiness of the equations from which the "calculated" numbers in this table are derived will be readily estimated upon inspection of the two columns which exhibit the difference between the calculated and observed numbers at the different years of age; and the substitution of the numerical values of the constants in equations (6) and (7) enables us to determine without difficulty the actual average age which corresponds to any given "age last birthday."

Making these numerical substitutions, the equations assume the form

$$-n + 102.507\ (0.85362)^n = -27.2949 + 0.01298027\ s_n \qquad (8)$$

$$t - 102.507\ (0.85362)^t = -27.2949 + 0.01298027\ s_{n+\frac{1}{2}} \qquad (9)$$

and yield at once the true ages corresponding to the average of the ages "at last birthday," which will be found as follows: —

Age last birthday.	Corresponding average age.
18	18.4814
23	23.4826
28	28.4843
33	33.4885
38	38.4924
43	43.4956
45	45.4968

Intermediate values may be found by interpolation with all needful accuracy.

Tables similar to Table II. prepared for each arm of the services independently, and for nine States or groups of States, and numbered as Tables III. to XIV. inclusive, are appended.

Such tables were originally constructed for a much larger number of groups; but these twelve will abundantly suffice to make manifest all the marked phenomena which the more detailed series has brought to light.

TABLE III.

United States Volunteer Infantry.

Age at last birthday.	Number at each year of age.	Proportion at and over specified age.		Difference. (C. — O.)	Proportion at each year of age.		Difference. (C. — O.)
		Observed.	Calculated.		Observed.	Calculated.	
13	113						
14	288						
15	636						
16	2053						
17	4653						
18	103420	10000	10000	0	1337	1252	− 85
19	71226	8663	8748	+ 85	921	1078	+157
20	56238	7742	7670	− 72	727	921	+194
21	75978	7015	6749	−266	983	802	−181
22	57485	6032	5947	− 85	743	694	− 49
23	48954	5289	5253	− 36	633	602	− 31
24	40852	4656	4651	− 5	528	524	− 4
25	36383	4128	4127	− 1	470	458	− 12
26	31292	3658	3669	+ 11	405	401	− 4
27	26369	3253	3268	+ 15	341	353	+ 12
28	27196	2912	2915	+ 3	352	312	− 40
29	18833	2560	2603	+ 43	244	276	+ 32
30	21937	2316	2327	+ 11	284	247	− 37
31	12814	2032	2080	+ 48	166	221	+ 55
32	17038	1866	1859	− 7	220	200	− 20
33	13678	1646	1659	+ 13	177	181	+ 4
34	12004	1469	1478	+ 9	155	166	+ 11
35	14558	1314	1312	− 2	188	152	− 36
36	10437	1126	1160	+ 34	135	141	+ 6
37	8782	991	1019	+ 28	114	131	+ 17
38	10025	877	888	+ 11	130	123	− 7
39	7200	747	765	+ 18	93	116	+ 23
40	10886	654	649	− 5	141	110	− 31
41	5634	513	539	+ 26	73	105	+ 32
42	8369	440	434	− 6	108	101	− 7
43	7900	332	333	+ 1	102	97	− 5
44	12274	230	236	+ 6	159	94	− 65
45	5509	71	142	+ 71	71	91	+ 20
46	737						
47	541						
48	532						
49	354						
50	1942						

TABLE IV.

United States Volunteer Cavalry.

Age at last birthday.	Number at each year of age.	Proportion at and over specified age.		Difference. (C. — O.)	Proportion at each year of age.		Difference. (C. — O.)
		Observed.	Calculated.		Observed.	Calculated.	
13	5						
14	15						
15	49						
16	232						
17	638						
18	15013	10000	10000	0	1295	1240	− 55
19	9767	8705	8760	+ 55	842	1074	+232
20	7864	7863	7686	−177	682	931	+249
21	12081	7181	6755	−426	1042	808	−234
22	9096	6139	5947	−192	784	703	− 81
23	7806	5355	5244	−111	673	612	− 61
24	6361	4682	4632	− 50	549	534	− 15
25	5724	4133	4098	− 35	494	467	− 27
26	4831	3639	3631	− 8	417	410	− 7
27	4192	3222	3221	− 1	360	360	0
28	4318	2862	2861	− 1	372	318	− 54
29	2845	2490	2543	+ 53	245	281	+ 36
30	3251	2245	2262	+ 17	280	250	− 30
31	2043	1965	2012	+ 47	176	223	+ 47
32	2450	1789	1789	0	211	200	− 11
33	1950	1578	1589	+ 11	168	180	+ 12
34	1679	1410	1410	0	145	163	+ 18
35	2130	1265	1247	− 18	184	148	− 36
36	1541	1081	1098	+ 17	133	135	+ 2
37	1268	948	963	+ 15	109	124	+ 15
38	1416	839	839	0	122	115	− 7
39	979	717	724	+ 7	84	107	+ 23
40	1441	633	618	− 15	124	100	− 24
41	822	509	518	+ 8	71	94	+ 23
42	1199	438	426	− 12	103	89	− 14
43	1079	335	337	+ 2	93	85	− 8
44	1851	242	252	+ 10	160	81	− 79
45	954	82	170	+ 88	82	78	− 4
46	105						
47	74						
48	73						
49	60						
50	203						

TABLE V.

United States Volunteer Artillery.

Age at last birthday.	Number at each year of age.	Proportion at and over specified age.		Difference. (C. — O.)	Proportion at each year of age.		Difference. (C. — O.)
		Observed.	Calculated.		Observed.	Calculated.	
14	2						
15	21						
16	61						
17	226						
18	5400	10000	10000	0	1275	1179	− 96
19	3439	8725	8821	+ 96	812	1024	+212
20	2627	7913	7797	−116	620	891	+271
21	4416	7293	6906	−387	1042	776	−266
22	3107	6251	6130	−121	734	678	− 56
23	2759	5517	5452	− 65	651	593	− 58
24	2163	4866	4859	− 7	511	521	+ 10
25	2012	4355	4338	− 17	475	459	− 16
26	1768	3880	3879	− 1	417	405	− 12
27	1505	3463	3474	+ 11	355	359	+ 4
28	1525	3108	3115	+ 7	360	320	− 40
29	1087	2748	2795	+ 47	257	286	+ 29
30	1213	2491	2509	+ 18	286	257	− 29
31	796	2205	2252	+ 47	188	232	+ 44
32	931	2017	2020	+ 3	220	211	− 9
33	753	1797	1809	+ 12	178	193	+ 15
34	724	1619	1616	− 3	171	177	+ 6
35	836	1448	1439	− 9	197	163	− 34
36	702	1251	1276	+ 25	166	151	− 15
37	477	1085	1125	+ 40	113	142	+ 29
38	579	972	983	+ 11	137	133	− 4
39	416	835	850	+ 15	98	126	+ 28
40	649	737	724	− 13	153	119	− 34
41	320	584	605	+ 21	76	114	+ 38
42	535	508	491	− 17	126	109	− 17
43	533	382	382	0	126	105	− 21
44	796	256	277	+ 21	188	102	− 86
45	289	68	175	+107	68	100	+ 32
46	45						
47	34						
48	31						
49	17						
50	68						

TABLE VI.

Ages of Maine, New Hampshire, Vermont, and Connecticut Vol's.

Age at last birthday.	Number at each year of age.	Proportion at and over specified age.		Difference. (C. — O.)	Proportion at each year of age.		Difference. (C. — O.)
		Observed.	Calculated.		Observed.	Calculated.	
13	3						
14	10						
15	27						
16	95						
17	223						
18	11694	10000	10001	+ 1	1522	1245	−277
19	6541	8478	8756	+278	852	1071	+219
20	5311	7626	7685	+ 59	691	923	+232
21	7477	6935	6762	−173	976	800	−176
22	5356	5959	5962	+ 3	699	685	− 14
23	4614	5260	5277	+ 17	604	598	− 6
24	3824	4656	4679	+ 23	500	519	+ 19
25	3357	4156	4160	+ 4	440	453	+ 13
26	2988	3716	3707	− 9	390	397	+ 7
27	2590	3326	3310	− 16	338	350	+ 12
28	2762	2988	2960	− 28	361	307	− 54
29	1881	2627	2653	+ 26	245	273	+ 28
30	1983	2382	2380	− 2	259	243	− 16
31	1362	2123	2137	+ 14	177	218	+ 41
32	1609	1946	1919	− 27	210	196	− 14
33	1427	1736	1723	− 13	185	178	− 7
34	1141	1551	1545	− 6	149	163	+ 14
35	1355	1402	1382	− 20	176	149	− 27
36	1046	1226	1233	+ 7	136	138	+ 2
37	989	1090	1095	+ 5	127	128	+ 1
38	1005	963	967	+ 4	131	118	− 13
39	817	832	849	+ 17	107	115	+ 8
40	969	725	734	+ 9	127	108	− 19
41	604	598	626	+ 28	77	102	+ 25
42	882	521	524	+ 3	115	97	− 18
43	870	406	427	+ 21	113	95	− 18
44	1789	293	332	+ 39	233	90	−143
45	459	60	242	+182	60	88	+ 28
46	50						
47	38						
48	34						
49	23						
50 & over.	60						

TABLE VII.

Ages of Massachusetts Volunteers.

Age at last birthday.	Number at each year of age.	Proportion at and over specified age.		Difference. (C. — O.)	Proportion at each year of age.		Difference. (C. — O.)
		Observed.	Calculated.		Observed.	Calculated.	
12	4						
13	4						
14	26						
15	44						
16	101						
17	289						
18	6894	10000	10000	0	1269	1145	−124
19	4582	8731	8855	+124	846	1002	+156
20	3604	7885	7853	− 32	666	877	+211
21	5429	7219	6976	−243	1003	771	−232
22	3860	6216	6205	− 11	713	678	− 35
23	3203	5513	5527	+ 14	592	597	+ 5
24	2871	4921	4930	+ 9	530	528	− 2
25	2474	4391	4402	+ 11	457	467	+ 10
26	2232	3934	3935	+ 1	412	415	+ 3
27	1962	3522	3520	− 2	362	370	+ 8
28	2041	3160	3150	− 10	377	330	− 47
29	1411	2783	2820	+ 37	260	296	+ 36
30	1564	2523	2524	+ 1	288	267	− 21
31	988	2235	2257	+ 22	183	242	+ 59
32	1233	2042	2015	− 27	228	219	− 9
33	1041	1814	1796	− 18	193	200	+ 7
34	980	1621	1596	− 25	181	184	+ 3
35	1213	1440	1412	− 28	224	169	− 55
36	761	1216	1243	+ 27	141	157	+ 16
37	699	1075	1086	+ 11	129	146	+ 17
38	828	946	940	− 6	153	137	− 16
39	600	793	803	+ 10	111	129	+ 18
40	838	682	674	− 8	155	122	− 33
41	440	527	552	+ 25	81	116	+ 35
42	658	446	436	− 10	122	110	− 12
43	596	324	326	+ 2	110	106	− 4
44	859	214	220	+ 6	159	102	− 57
45	296	55	118	+ 63	55	98	+ 43
46	28						
47	14						
48	16						
49	9						
50 & over.	33						

TABLE VIII.

Ages of New York Volunteers.

Age at last birthday.	Number. at each year of age.	Proportion at and over specified age.		Difference. (C. — O.)	Proportion at each year of age.		Difference. (C. — O.)
		Observed.	Calculated.		Observed.	Calculated.	
13	17						
14	63						
15	153						
16	448						
17	699						
18	19737	10000	10000	0	1087	1173	+ 86
19	16233	8913	8827	− 86	894	1019	+125
20	11286	8019	7808	−211	621	887	+266
21	20227	7398	6922	−476	1114	773	−341
22	13689	6284	6149	−135	754	675	− 79
23	11516	5530	5774	+244	634	592	− 42
24	9488	4896	4882	− 14	523	520	− 3
25	8648	4373	4363	− 10	476	459	− 17
26	7285	3897	3904	+ 7	401	406	+ 5
27	6223	3496	3498	+ 2	343	360	+ 17
28	6652	3153	3138	− 15	366	322	− 44
29	4552	2787	2816	+ 29	251	289	+ 38
30	5474	2536	2527	− 9	301	260	− 41
31	3287	2235	2267	+ 32	181	236	+ 55
32	4533	2054	2031	− 23	249	215	− 34
33	3330	1805	1816	+ 11	184	197	+ 13
34	3135	1621	1619	− 2	173	182	+ 9
35	3885	1448	1437	− 11	114	168	+ 54
36	2872	1234	1269	+ 35	158	157	− 1
37	2201	1076	1112	+ 36	121	146	+ 25
38	2709	955	966	+ 11	149	139	− 10
39	1858	806	827	+ 21	103	132	+ 29
40	3157	703	695	− 8	173	126	− 47
41	1268	530	569	+ 39	70	121	+ 51
42	2302	460	448	− 12	127	116	− 11
43	2068	333	332	− 1	114	112	− 2
44	3148	219	220	+ 1	173	109	− 64
45	831	46	111	+ 65	46	106	+ 60
46	87						
47	41						
48	53						
49	23						
50 & over.	103						

TABLE IX.

Ages of Pennsylvania Volunteers (including Reserves).

Age at last birthday.	Number at each year of age.	Proportion at and over specified age.		Difference. (C. — O.)	Proportion at each year of age.		Difference. (C. — O.)
		Observed.	Calculated.		Observed.	Calculated.	
13	23						
14	51						
15	85						
16	241						
17	486						
18	13052	10000	10000	0	1137	1339	+202
19	11410	8863	8661	−202	994	1131	+137
20	8234	7869	7530	−339	717	959	+242
21	13336	7152	6571	−581	1161	814	−347
22	9376	5991	5757	−234	816	694	−122
23	7696	5175	5063	−112	670	595	− 75
24	6061	4505	4468	− 37	528	510	− 18
25	5375	3977	3958	− 19	468	441	− 27
26	4420	3509	3517	+ 8	385	382	− 3
27	3576	3124	3135	+ 11	311	334	+ 23
28	3817	2813	2801	− 12	332	293	− 39
29	2644	2481	2508	+ 27	230	260	+ 30
30	2926	2251	2248	− 3	255	232	− 23
31	2029	1996	2016	+ 20	177	208	+ 31
32	2375	1819	1808	− 11	207	188	− 19
33	1903	1612	1620	+ 8	166	173	+ 7
34	1657	1446	1447	+ 1	144	158	+ 14
35	2089	1302	1289	− 13	182	147	− 35
36	1490	1120	1142	+ 22	130	138	+ 8
37	1290	990	1004	+ 14	112	130	+ 18
38	1434	878	874	− 4	125	124	− 1
39	1141	753	750	− 3	99	118	+ 19
40	1692	654	632	− 22	147	113	− 34
41	918	507	519	+ 12	80	109	+ 29
42	1431	427	410	+ 17	124	106	− 18
43	1318	303	307	+ 4	115	103	− 12
44	1674	188	206	+ 18	146	101	− 45
45	480	42	105	+ 63	42	99	+ 57
46	73						
47	46						
48	49						
49	36						
50 & over.	109						

TABLE X.

Ages of Ohio Volunteers.

Age at last birthday.	Number at each year of age.	Proportion at and over specified age.		Difference. (C. — O.)	Proportion at each year of age.		Difference. (C. — O.)
		Observed.	Calculated.		Observed.	Calculated.	
13	21						
14	44						
15	103						
16	470						
17	1476						
18	23495	10000	10000	0	1567	1359	−208
19	14986	8433	8641	+208	999	1143	+144
20	12358	7434	7498	+ 64	825	963	+138
21	12819	6609	6535	− 74	855	815	− 40
22	10499	5754	5720	− 34	700	692	− 8
23	9297	5054	5028	− 26	620	590	− 30
24	7327	4434	4438	− 6	489	505	+ 16
25	6502	3945	3933	− 12	430	435	+ 5
26	5678	3515	3498	− 17	382	377	− 5
27	4739	3133	3121	− 12	316	329	+ 13
28	4997	2817	2792	− 25	333	289	− 44
29	3570	2484	2503	+ 19	238	256	+ 18
30	3960	2246	2247	+ 1	264	228	− 36
31	2596	1982	2019	+ 37	174	206	+ 32
32	3029	1808	1813	+ 5	201	187	− 14
33	2669	1607	1626	+ 19	178	171	− 7
34	2302	1429	1455	+ 26	154	159	+ 5
35	2659	1275	1296	+ 21	178	148	− 30
36	2216	1097	1148	+ 51	147	139	− 8
37	1830	950	1009	+ 59	123	132	+ 9
38	1959	827	877	+ 50	130	125	− 5
39	1424	697	752	+ 55	95	120	+ 25
40	1880	602	632	+ 30	126	116	− 10
41	1097	476	516	+ 40	73	113	+ 40
42	1513	403	403	0	101	110	+ 9
43	1337	302	293	− 9	89	108	+ 19
44	2070	213	185	− 28	138	106	− 32
45	1128	75	79	+ 4	75	104	+ 29
46	202						
47	161						
48	145						
49	104						
50 & over.	471						

TABLE XI.

Ages of Indiana Volunteers.

Age at last birthday.	Number at each year of age.	Proportion at and over specified age.		Difference. (C. — O.)	Proportion at each year of age.		Difference. (C. — O.)
		Observed.	Calculated.		Observed.	Calculated.	
13	13						
14	16						
15	39						
16	162						
17	578						
18	11178	10000	10000	0	1608	1446	−162
19	7175	8392	8554	+162	1032	1223	+191
20	6478	7360	7331	− 29	932	1035	+103
21	6398	6428	6296	−132	920	877	− 43
22	5580	5508	5419	− 89	802	744	− 58
23	4562	4706	4675	− 31	656	632	− 24
24	3782	4050	4043	− 7	544	538	− 6
25	3216	3506	3505	− 1	462	460	− 2
26	2707	3044	3045	+ 1	390	394	+ 4
27	2269	2654	2651	− 3	326	337	+ 11
28	2272	2328	2314	− 14	327	290	− 37
29	1513	2001	2024	+ 23	217	251	+ 34
30	1799	1784	1773	− 11	259	218	− 41
31	1013	1525	1555	+ 30	145	190	+ 45
32	1230	1380	1365	− 15	177	166	− 11
33	1046	1203	1200	− 3	151	146	− 5
34	871	1052	1053	+ 1	125	130	+ 5
35	962	927	923	− 4	138	116	− 22
36	666	789	806	+ 17	96	104	+ 8
37	589	693	702	+ 9	85	94	+ 9
38	656	608	608	0	94	86	− 8
39	428	514	522	+ 8	62	79	+ 17
40	683	452	443	− 9	98	73	− 25
41	371	354	370	+ 16	53	68	+ 15
42	482	301	302	+ 1	69	64	− 5
43	471	232	238	+ 6	68	60	− 8
44	682	164	178	+ 14	98	57	− 41
45	457	66	121	+ 55	66	55	− 11
46	70						
47	37						
48	50						
49	24						
50 & over.	146						

TABLE XII.

Ages of Michigan Volunteers.

Age at last birthday.	Number at each year of age.	Proportion at and over specified age.		Difference. (C. — O.)	Proportion at each year of age.		Difference. (C. — O.)
		Observed.	Calculated.		Observed.	Calculated.	
13	3						
14	9						
15	27						
16	112						
17	299						
18	5862	10000	10000	0	1523	1279	−244
19	3437	8477	8721	+244	893	1098	+205
20	2767	7584	7623	+ 39	719	943	+224
21	3727	6865	6680	−185	968	812	−156
22	2802	5897	5868	− 29	728	700	− 28
23	2337	5169	5168	− 1	607	605	− 2
24	1963	4562	4563	+ 1	510	524	+ 14
25	1724	4052	4039	− 13	448	455	+ 7
26	1568	3604	3584	− 20	407	396	− 11
27	1297	3197	3188	− 9	337	346	+ 9
28	1335	2860	2842	− 18	347	304	− 43
29	923	2513	2538	+ 25	240	268	+ 28
30	989	2273	2270	− 3	257	237	− 20
31	695	2016	2033	+ 17	180	211	+ 31
32	843	1836	1822	− 14	219	188	− 31
33	614	1617	1634	+ 17	160	169	+ 9
34	527	1457	1465	+ 8	137	153	+ 16
35	668	1320	1312	− 8	173	140	− 33
36	481	1147	1172	+ 25	125	128	+ 3
37	411	1022	1044	+ 22	107	118	+ 11
38	458	915	926	+ 11	119	109	− 10
39	313	796	817	+ 21	81	102	+ 21
40	466	715	715	+ 0	121	96	− 25
41	256	594	619	+ 25	67	91	+ 24
42	403	527	528	+ 1	105	86	− 19
43	400	422	442	+ 20	104	83	− 21
44	825	318	359	+ 41	214	79	−135
45	398	104	280	+176	104	77	− 27
46	44						
47	23						
48	26						
49	14						
50 & over.	61						

TABLE XIII.

Ages of Illinois Volunteers.

Age at last birthday.	Number at each year of age.	Proportion at and over specified age.		Difference. (C. — O.)	Proportion at each year of age.		Difference. (C. — O.)
		Observed.	Calculated.		Observed.	Calculated.	
13	5						
14	23						
15	65						
16	250						
17	539						
18	10167	10000	10080	+ 80	1070	942	−128
19	8348	8930	9138	+208	879	1043	+164
20	7076	8051	8095	+ 44	745	958	+213
21	8709	7306	7137	−169	916	858	− 58
22	7441	6390	6279	−111	783	766	− 17
23	6872	5607	5513	− 94	723	677	− 46
24	6019	4884	4836	− 48	634	600	− 34
25	5315	4250	4236	− 14	559	529	− 30
26	4441	3691	3707	+ 16	468	465	− 3
27	3810	3223	3242	+ 19	401	410	+ 9
28	3677	2822	2832	+ 10	387	358	− 29
29	2622	2435	2474	+ 39	276	315	+ 39
30	2869	2159	2159	0	302	276	− 26
31	1847	1857	1883	+ 26	194	242	+ 48
32	2076	1663	1641	− 22	219	211	− 8
33	1666	1444	1430	− 14	175	185	+ 10
34	1508	1269	1245	− 24	159	162	+ 3
35	1568	1110	1083	− 27	165	142	− 23
36	1243	945	941	− 4	131	124	− 7
37	944	814	817	+ 3	99	110	+ 11
38	1056	715	707	− 8	111	96	− 15
39	725	604	611	+ 7	77	87	+ 10
40	1040	527	524	− 3	109	77	− 32
41	607	418	447	+ 29	64	69	+ 5
42	816	354	378	+ 24	86	64	− 18
43	734	268	314	+ 46	77	59	− 18
44	1075	191	255	+ 69	113	54	− 59
45	737	78	201	+123	78	50	− 28
46	88						
47	86						
48	78						
49	45						
50 & over.	237						

TABLE XIV.

Ages of Wisconsin and Iowa Volunteers.

Age at last birthday.	Number at each year of age.	Proportion at and over specified age.		Difference. (C. — O.)	Proportion at each year of age.		Difference. (C. — O.)
		Observed.	Calculated.		Observed.	Calculated.	
13	11						
14	22						
15	79						
16	369						
17	829						
18	11083	10000	10000	0	1485	1221	−264
19	6440	8515	8779	+264	863	1048	+185
20	4874	7652	7731	+ 79	653	902	+249
21	7082	6999	6829	−170	949	778	−171
22	5271	6050	6050	0	707	673	− 34
23	4240	5343	5377	+ 34	569	585	+ 16
24	3718	4774	4792	+ 18	499	510	+ 11
25	3260	4275	4282	+ 7	437	447	+ 10
26	2953	3838	3835	− 3	396	393	− 3
27	2675	3442	3442	0	359	345	− 14
28	2495	3083	3097	+ 14	334	310	− 24
29	1844	2749	2787	+ 38	247	277	+ 30
30	1973	2502	2510	+ 8	264	250	− 14
31	1472	2238	2260	+ 22	196	227	+ 31
32	1674	2042	2033	− 9	224	207	− 17
33	1432	1818	1826	+ 8	192	191	− 1
34	1237	1626	1635	+ 9	166	177	+ 11
35	1359	1460	1458	− 2	182	165	− 17
36	1154	1278	1293	+ 15	155	155	0
37	1022	1123	1138	+ 15	137	146	+ 9
38	1104	986	992	+ 6	148	139	− 9
39	873	838	853	+ 15	117	133	+ 16
40	967	721	720	− 1	130	128	− 2
41	670	591	592	+ 1	90	124	+ 34
42	886	501	468	− 33	119	120	+ 1
43	950	382	348	− 34	127	117	− 10
44	1374	255	231	− 24	184	114	− 70
45	531	71	117	+ 46	71	112	+ 41
46	113						
47	108						
48	115						
49	76						
50 & over.	632						

The agreement of these several special results with those deduced from their aggregate is remarkable. Only in one case, that of the Illinois troops, has the simple formula

$$s_n = a - bn + ch^n$$

failed to give all desired accordance between theory and observation; and throughout the whole series the same peculiarities in the residuals are recognizable. In this connection I may add, what is in itself very significant, that attempts to deduce a law of distribution of age for troops recruited in Missouri, Kentucky, Tennessee, and Virginia have proved fruitless, and only small success was attainable for the Maryland volunteers. The inference is obvious, that the volunteering of troops from these States was not subject to the undisturbed influence of any statistical law. In the case of Illinois troops, a curious anomaly manifested itself in the residuals, namely, a cyclical or periodic term. This was found to be represented with sufficient accuracy by adding to the formula a term $d \sin \sqrt{n} \cdot 68°$, in which $d = 314$. I know of no satisfactory interpretation of this expression, but it has been used in the preparation of the table for that State.

In Table XV. is presented a summary of the results deduced from the special groups presented in Tables III. to XIV. All the constants are reduced to the same scale, and hold good for 10 000 troops of the ages 18 to 45 at last birthday, inclusive. The mean ages, as here given, refer, not to the last birthday, but to the actual date of enlistment.

The values of the constants for these special tables have been determined from a smaller number of equations of condition than were used for the grand total. In that each year was specially used; in these the results were deduced from eight normal places.

TABLE XV.

Constants deduced for Special Classes of Volunteers.

Class.	Number of Soldiers		Mean age at enlistment.		$a.$	$b.$	$c.$	$h.$
	Of all ages.	Of military age.	For all.	For 18 to 45.				
Total Enlisted Men	1012273	996647	25.8362	25.8083	2102.8	77.04	7897.2	0.8536
Total Infantry	785120	773271	25.7827	25.7484	2080.0	75.84	7920.0	0.8514
Total Cavalry	117405	115951	25.8110	25.7795	1595.0	57.90	8405.0	0.8593
Total Artillery	42862	42357	26.1576	26.1202	2239.0	81.20	7761.0	0.8585
Me., N.H., Vt., Conn.	76445	75881	25.8792	25.8423	2112.0	73.06	7889.0	0.8514
Massachusetts	54705	54137	26.0561	26.0943	2016.0	76.40	7984.0	0.8662
New York	183281	181594	26.1308	26.1642	2390.5	88.86	7609.5	0.8575
Pennsylvania	116043	114844	25.8227	25.8331	2477.4	90.20	7523.0	0.8340
Ohio	153133	149936	25.4936	25.3859	2625.0	96.08	7375.0	0.8287
Indiana	70673	69536	24.7100	24.6858	1175.0	42.18	8825.0	0.8409
Michigan	39107	38489	25.5290	25.5276	1827.0	61.30	8173.0	0.8510
Illinois	96409	95003	25.9369	25.8935	2023.0	70.66	8057.0	0.8558
Wisconsin and Iowa	76987	74613	26.1571	25.9991	2737.0	100.20	7263.0	0.8456

In considering the residuals, the most striking feature is the excess of the recorded numbers at 18 and 21, which latter excess is counterbalanced by a deficiency at 20 and to some extent at 19 also. The explanation of this is readily found in the facts that enlistments of youths under 18 are not valid without the formal consent of parents, and that 21 is the period at which minority ceases. There can be no reasonable doubt that these residuals furnish the measure of the number under 18 and under 21, who misstated their age to the mustering officer. At the age of 18 the discordance is less marked than at 21, since the inducements to misstate operated near this age in different directions, many of those at 18 probably representing themselves as 21 years old, while their number was made good by others who untruly declared themselves as having completed their 18th year.

The excess of the recorded number at 21 averages $1\frac{4}{5}$ per cent., that deficiency at 20 is about 2 per cent., and at 19 about $1\frac{3}{5}$ per cent. The number recorded for 18 years is in excess by 1 per cent., although it varies very considerably in the different groups.

A large excess, representing the number of those who from similar motives understated their ages, is also to be seen at the age

of 44 in most States, corresponding to an analogous deficiency at 45. This varies, however, in different States, owing in all probability to the different interpretation by the mustering officers of that provision of the law which precluded the acceptance of men over 45 years old. The average, in the more elaborately calculated table for the grand total, places the number at 44 in excess of the computed number by two thirds of its whole amount, and leaves that at 45 in defect by one fourth part.

For all other ages than those enumerated, the regular excess or defect of the residuals furnishes apparently the measure of the accuracy with which the ages were stated or recorded. It will be seen that at those ages which correspond to what are called round numbers, such as those divisible by 10, also, though to a less extent, at those divisible by 5, and in a still less but yet recognizable degree, at those divisible by 2, the recorded numbers are in excess; while the adjacent numbers, especially those ending in 1, 9, and 7, are in defect. The natural tendency which every one will recognize, and which inclines us to make use of certain more habitually employed numbers, rather than to use a minuteness repugnant to some persons, furnishes an adequate and, as I believe, the true explanation.

It will be readily noted that where any two of the above-named principles conflict, the residual is diminished; and that where they act in combination it is increased.

Lines showing the computed and the enrolled numbers of enlisted men are given on Chart *A*, and readily manifest these facts to the eye. The other data upon this chart are given for comparison, and will be referred to hereafter. It will be borne in mind that the numbers given do not, by a large amount, represent the actual numbers of enlisted volunteers or of volunteer officers, nor probably so much as two fifths of the total number of our soldiers in the struggle for national existence. They are relative quantities, deduced from only those data cited at the commencement of this paper, and illustrate, not the actual numbers for our troops, but the relative distribution of their ages.

The same results are presented in another form upon Chart *B*, which exhibits, for the enlisted men, the officers, and the white male population, the *proportion at and over the specified ages* and under 45 years, for each 10 000 men of military age.

Charts *C* and *D* show the law by which the ratios of officers and enlisted men to the white male population vary with the age. All the numbers are reduced to the scale of ten thousand of population at 18 years, Chart *C* being constructed in reference to the whole United States, and Chart *D* to the Loyal States only.

3. *Ages of Officers.*

The total number of officers of all ages is 37184, that of those between 18 and 46 being 35953.

On comparing the numbers at the several ages with the formula

$$s_n = a - bn + ch^n$$

we find at once that for certain ages the value of h would be an impossible one; and that for other years, which would yield possible values, these values are so discordant and the residuals to which they lead become so large that it is manifest that the curve can be represented neither by this nor by any similar law.

Many trials have led to the empirical formula

$$s_n = a - bn^{k'} + c \sin n^k \theta \qquad \text{(A)}$$

as that which best represents the character of the curve. The extremely complicated manner, however, in which the six constants of this equation enter into the formula renders the attainment of a solution from six equations, by any direct process, a matter of great difficulty and inconvenience. Of course the constant a represents the value of s_n for $n = 0$, so that the problem really consists in the determination of the five quantities b, c, k', k, and θ. Graphic representations of the curve, by showing the points at which the third term becomes $= 0$, facilitated the approximate determination of these constants, and thus equations of condition were formed which have led to quite satisfactory values, giving an agreement between the formula and the observed numbers nearly if not quite as good as that obtained for the enlisted men by the formula already described.

Subsequently, investigations made for the purpose of extending this formula to the ages from 46 to 50 showed a deviation for these later years. This deviation seems only to be reconciled by the employment of an additional term containing two more constants,

and the term thus found proves applicable to all ages above 30, essentially diminishing the residuals for all subsequent years.

The formula then stands for each 10 000 officers

$$s_n = 10\,000 - 736\,n^{0.75} + 1259 \sin n^{0.536} \times 45°.64 + 100 \sin (n - 12)\,18°$$

in which the last term is only to be employed for positive values of $n - 12$, that is, for ages above 30 years.

The near agreement of this formula with the observations will be recognized on Table XVI., which exhibits for each year of age from 18 to 50, as well as for those above 50, the actual and the proportionate observed number of officers, both at, and at and over, the given age, together with the corresponding numbers as deduced from the formula, and the discordances between Computation and Observation.

The dissimilarity of the curves thus found for officers and for enlisted men is most striking, as will be perceived by reference to Charts *A* and *B*. The chief discordance for the officers' curve is for the age of 18 years, at which, or at 19, the formula seems to fail. This is probably due in part to the fact that comparatively few officers were commissioned under the age of legal maturity, so that the law governing the distribution by age ought not to be regarded as applicable below 21 years.

TABLE XVI.

Ages of Officers of United States Volunteers.

Age at last birthday.	Number at given age.	Proportion at given age			Number at and over given age.	Proportion at and over given age		
		Observed.	Calculated.	Difference. (C. — O.)		Observed.	Calculated.	Difference. (C. — O.)
13								
14								
15	1				37183			
16	5				37182			
17	5				37177			
18	178	48	−164	−212	37172	10000	10000	0
19	409	110	+233	+123	37094	9952	10164	+212
20	687	185	351	+166	36685	9842	9931	+ 89
21	1630	439	443	+ 4	35998	9657	9580	− 77
22	1839	495	500	+ 5	34368	9218	9137	− 81
23	2101	565	537	− 28	32529	8723	8637	− 86
24	2234	601	557	− 44	30428	8158	8100	− 58
25	2161	581	567	− 14	28194	7557	7543	− 14
26	2114	569	563	− 6	26033	6976	6976	0
27	1968	529	555	+ 26	23919	6407	6413	+ 6
28	2071	557	536	− 21	21951	5878	5858	− 20
29	1756	472	516	+ 44	19880	5321	5322	+ 1
30	1836	494	457	− 37	18124	4849	4806	− 43
31	1429	384	430	+ 46	16288	4355	4349	− 6
32	1613	434	405	− 29	14859	3971	3919	− 52
33	1422	383	381	− 2	13246	3537	3514	− 23
34	1324	356	359	+ 3	11824	3154	3133	− 21
35	1434	386	335	− 51	10500	2798	2774	− 24
36	1221	328	313	− 15	9066	2412	2439	+ 27
37	1031	277	291	+ 14	7845	2084	2126	+ 42
38	1033	278	269	− 9	6814	1807	1835	+ 28
39	813	219	245	+ 26	5781	1529	1566	+ 37
40	874	235	222	− 13	4968	1310	1321	+ 11
41	557	149	197	+ 48	4094	1075	1099	+ 24
42	656	176	171	− 5	3537	926	902	− 24
43	485	130	148	+ 18	2881	750	731	− 19
44	598	161	124	− 37	2396	620	583	− 37
45	478	130	100	− 30	1798	459	459	0
46	217	58	86	+ 28	1320	329	359	+ 30
47	184	50	70	+ 20	1103	271	273	+ 2
48	175	47	58	+ 11	919	221	203	− 18
49	121	33	55	+ 22	744	174	145	− 29
50 & over.	523	141	90	− 51	523	141	90	− 51

The mean age at last birthday for all the officers is . $30^{y}.4406$
" " " for those between 18 & 45 29.8338
and the mean age of the mean at last birthday is . 29.45
or about 29.94 at the time of their muster into the service. Above and below this age the number of officers was equal.

The annexed Table XVII. exhibits the relative proportions of officers to the enlisted men, and of these to the white male population of the whole United States and of the Loyal States respectively, as given by the census of 1860, taken less than one year before the call to arms.

The caution must here be repeated, that the "proportion of enlisted men to the population," as here given, does not at all apply to the armies of the nation during the rebellion. It relates solely to the number of volunteer troops here considered; and this Table XVII. is presented solely to make manifest the laws according to which the ratios of enlisted men to the population, and the ratios of officers to men, varied with the age.

TABLE XVII.

Relative Proportions of Officers, Enlisted Men, and White Male Population, for the first million of Volunteers.

AGE.	Proportion of Officers to Enlisted Men.	Proportion of Enlisted Men to Population of U. States.	Proportion of Enlisted Men to Population of Loyal States.	AGE.	Proportion of Officers to Enlisted Men.	Proportion of Enlisted Men to Population of U. States.	Proportion of Enlisted Men to Population of Loyal States.
18	0.001	0.448	0.570	32	0.072	0.100	0.128
19	0.007	0.393	0.502	33	0.074	0.093	0.119
20	0.013	0.345	0.442	34	0.076	0.088	0.112
21	0.019	0.305	0.391	35	0.077	0.084	0.105
22	0.025	0.269	0.363	36	0.078	0.080	0.100
23	0.031	0.239	0.308	37	0.077	0.077	0.096
24	0.037	0.212	0.275	38	0.075	0.075	0.093
25	0.043	0.190	0.246	39	0.073	0.074	0.090
26	0.048	0.170	0.221	40	0.068	0.073	0.088
27	0.054	0.154	0.199	41	0.062	0.073	0.088
28	0.059	0.139	0.180	42	0.057	0.073	0.087
29	0.063	0.127	0.164	43	0.049	0.074	0.087
30	0.065	0.116	0.150	44	0.041	0.075	0.088
31	0.068	0.107	0.138	45	0.033	0.076	0.089

4. *Population of the United States and of the Loyal States.*

The great and unexpected dissimilarity between the law of distribution of age for officers and for men led, as I have already mentioned, to an investigation of the ages of the white male population, both of the whole United States, and of the Loyal States considered by themselves. And, in the absence of any distinct criterion, those States which were free from slavery in 1860, together with Delaware, Maryland, Kentucky, and Missouri, have been classed as Loyal States. The territory of West Virginia, eastern Tennessee, &c., is thus excluded, although inhabited by a thoroughly loyal population, which contributed about twenty of the regiments here computed; and about ten other regiments, included in our data, were raised in States not accounted loyal. But all these are offset by the very considerable portion of the inhabitants of the four Slave States above named, from which the insurgent army was reinforced.

The only materials available for the inquiry are contained in the tables, derived from the official census of the United States in 1860. Of course it is the male population alone which has any relation to the present research.

The difficulty of deducing from these meagre details the number of males at each year of military age is apparent at the first glance. Had the classification between the ages of 20 and 50 been in six groups of five years each, instead of three groups of ten years, the facility and accuracy of the investigation would have been incomparably greater. As it is, the only available data are contained in the second column of the following tables, XVIII. and XIX. These tables give, in column 9, the results of the formulas obtained for representing the observed numbers given in column 2. The degree of correctness of these formulas may be estimated by means of column 4, which shows the excess of the calculated number over the number given by the census, in decimals of the latter. The accordance for ages above 20 years is remarkably good. Beyond 50 years the agreement is not so close as between 20 and 50, but is nevertheless quite tolerable; but the comparison is omitted here as not pertinent to the subject, since none of the census-numbers for groups of ages above 50 have been employed in the computation.

The other columns require no explanation. It will be remembered that the numbers of enlisted men and officers here given are merely those belonging to the original volunteer regiments at the time of their enlistment, excluding all recruits, substitutes, drafted men, &c. Also, that the numbers apply only to those regiments which had been mustered into the United States service prior to the collection of our data, as shown on page 2.

TABLE XVIII.

White Male Population of the United States in 1860.

Comparison between Computed and Observed Ages.

Age at last birthday.	White Male Population of the United States.		Difference. (C. — O.)	Enlisted Men of first volunteers.	Officers of first volunteers.	Ratio to White Male Population.	
	Census.	Computed.				Men.	Officers.
10 — 15	1 578 274	1 547 730	−0.0193				
15 — 20	1 391 950	1 422 340	+0.0245				
18 — 20		553 360		219 200	587	0.395	0.0011
20 — 30	2 465 276	2 436 770	−0.0116	529 809	18 561	0.217	0.0076
30 — 40	1 847 259	1 847 810	0.0000	165 292	13 156	0.090	0.0071
40 — 45		807 860		63 667		0.079	
40 — 50	1 215 031	1 216 690	+0.0014		4 868		0.0040
18 — 45		5 645 800		977 968			

TABLE XIX.

White Male Population of the Loyal States in 1860.

Comparison between Computed and Observed Ages.

Age at last birthday.	White Male Population of the Loyal States.		Difference. (C. — O.)	Enlisted Men of first volunteers.	Officers of first volunteers.	Ratio to White Male Population.	
	Census.	Computed.				Men.	Officers.
10 — 15	1 211 521	1 179 260	−0.0266				
15 — 20	1 095 934	1 110 770	+0.0135				
18 — 20		435 100		219 200	587	0.502	0.0014
20 — 30	1 971 486	1 956 890	−0.0075	529 809	18 561	0.271	0.0095
30 — 40	1 517 736	1 517 720	0.0000	165 292	13 156	0.109	0.0087
40 — 45		664 510		63 667		0.096	
40 — 50	996 481	996 350	0.0000		4 868		0.0049
18 — 45		4 574 220		977 968			

The formulas which thus represent the number of white males from the age of 10 years upwards are, —

for the United States

$$x = 445\,440 \sin (134° 34' + (y - 10) \cdot 52')$$

for the Loyal States

$$x = 257\,870 \sin (111° 6'.1 + (y - 10) \cdot 80'.2)$$

in which x is the number at the year of age y.

Assuming these values to be correct, we find the distribution of the white male population in 1860 to have been as represented in Tables XX. and XXI.

These tables show, for the United States and the Loyal States respectively, the actual numbers: — first, at each year of age from 15 to 50, inclusive; secondly, at and over each year of age from 15 to 50, inclusive; thirdly, at and over each year within the limits of military age from 18 upwards, and also the corresponding relative or proportional numbers, using those for 18 years as the units.

Subsequent investigation has led to the detection of a formula totally different in structure from those above given, but which, although its agreement with the census-numbers within the years of military age is by no means so close as these afford, yet represents the various censuses of the United States and those of foreign countries throughout the period of human life with a degree of precision never before attained, so far as I am aware. It represents the number of infants under one year as well as, and indeed better than, the number at middle life or advanced years; and I cannot avoid the conviction that this formula affords an important step toward the true mathematical expression of what we may call the life-curve. Modifications will doubtless be made in it; indeed, it manifestly gives the numbers too small for the ages under 5 years, over 70 years, and between 20 and 45 years, while those of later childhood and youth on the one side, and of advanced maturity on the other, are in excess. But the discordances are small, and I hardly think that any expression of equal simplicity will be found which will represent the life-curve more closely.

Of this formula, which is simply

$$s_n = a \sin n k^n \theta$$

where s_n represents the sum of all under the age n, a is the total number, and k, θ are two constants characteristic of the especial population under examination, details and applications are given in the Appendix; where also are tables exhibiting the distribution of the total white male population of the United States and of the Loyal States, as given by this law. The values differ slightly from those in Tables XXI. and XXII., which, for the census of 1860 at least, seem to be more accurate within the limits to which they are extended, although the corresponding numbers beyond these limits would be less accordant with observation.

TABLE XX.

White Male Population of the United States in 1860.

Age at last birthday.	Actual Number			Relative Number		
	At the given age.	At and over given age.	At and over given age and under 46.	At the given age.	At and over given age.	At and over given age & under 46.
15	294 770	8 252 612				
16	289 680	7 957 842				
17	284 530	7 668 162				
18	279 320	7 383 632	5 645 800	10 000	10 000	10 000
19	274 040	7 104 312	5 366 480	9 811	9 622	9 505
20	268 700	6 830 272	5 092 440	9 620	9 251	9 020
21	263 290	6 561 572	4 823 740	9 426	8 887	8 544
22	257 820	6 298 282	4 560 450	9 230	8 530	8 078
23	252 300	6 040 462	4 302 630	9 033	8 181	7 621
24	246 720	5 788 162	4 050 330	8 833	7 839	7 174
25	241 090	5 541 442	3 803 610	8 631	7 505	6 737
26	235 380	5 300 352	3 562 520	8 427	7 179	6 310
27	229 640	5 064 972	3 327 140	8 222	6 860	5 893
28	223 840	4 835 332	3 097 500	8 014	6 549	5 486
29	217 990	4 611 492	2 873 660	7 804	6 246	5 090
30	212 090	4 393 502	2 655 670	7 593	5 951	4 704
31	206 140	4 181 412	2 443 580	7 380	5 664	4 328
32	200 140	3 975 272	2 237 440	7 165	5 385	3 963
33	194 100	3 775 132	2 037 300	6 949	5 114	3 609
34	188 020	3 581 032	1 843 200	6 731	4 851	3 265
35	181 890	3 393 012	1 655 180	6 512	4 596	2 932
36	175 710	3 211 122	1 473 290	6 291	4 350	2 610
37	169 500	3 035 412	1 297 580	6 068	4 112	2 299
38	163 250	2 865 912	1 128 080	5 845	3 882	1 999
39	156 970	2 702 662	964 830	5 620	3 661	1 710
40	150 640	2 545 692	807 860	5 393	3 448	1 432
41	144 290	2 395 052	657 220	5 166	3 244	1 165
42	137 900	2 250 762	512 930	4 937	3 049	909
43	131 470	2 112 862	375 030	4 707	2 862	665
44	125 020	1 981 392	243 560	4 476	2 684	432
45	118 540	1 856 372	118 540	4 244	2 515	210
46	112 030	1 737 832		4 011	2 354	
47	105 500	1 625 802		3 777	2 202	
48	98 940	1 520 302		3 542	2 059	
49	92 360	1 421 362		3 307	1 925	
50	85 760	1 329 002		3 072	1 800	

TABLE XXI.

White Male Population of the Loyal States in 1860.

Age at last birth-day.	Actual Number			Relative Number			Prop. to W. Male Pop. of United States.
	At the given age.	At and over given age.	At and over given age and under 46.	At the given age.	At and over given age.	At and over given age & under 46.	
15	228 120	6 675 533					7 739
16	225 270	6 447 413					7 776
17	222 280	6 222 143					7 812
18	219 160	5 999 863	4 574 220	10 000	10 000	10 000	7 846
19	215 940	5 780 703	4 355 060	9 853	9 634	9 521	7 880
20	212 600	5 564 763	4 139 120	9 700	9 275	9 049	7 912
21	209 130	5 352 163	3 926 520	9 542	8 920	8 584	7 943
22	205 550	5 143 033	3 717 390	9 379	8 572	8 127	7 973
23	201 870	4 937 483	3 511 840	9 211	8 229	7 678	8 001
24	198 070	4 735 613	3 309 970	9 038	7 893	7 237	8 028
25	194 160	4 537 543	3 111 900	8 859	7 563	6 804	8 054
26	190 150	4 343 383	2 917 740	8 676	7 239	6 379	8 078
27	186 040	4 153 233	2 727 590	8 488	6 922	5 963	8 101
28	181 820	3 967 193	2 541 550	8 296	6 612	5 555	8 123
29	177 500	3 785 373	2 359 730	8 099	6 309	5 157	8 141
30	173 100	3 607 873	2 182 230	7 898	6 013	4 769	8 162
31	168 590	3 434 773	2 009 130	7 692	5 725	4 391	8 179
32	163 990	3 266 183	1 840 540	7 483	5 444	4 022	8 194
33	159 300	3 102 193	1 676 550	7 269	5 170	3 663	8 207
34	154 530	2 942 893	1 517 250	7 051	4 905	3 315	8 219
35	149 680	2 788 363	1 362 720	6 829	4 647	2 977	8 229
36	144 730	2 638 683	1 213 040	6 604	4 398	2 650	8 237
37	139 720	2 493 953	1 068 310	6 375	4 157	2 334	8 243
38	134 620	2 354 233	928 590	6 143	3 924	2 029	8 246
39	129 460	2 219 613	793 970	5 907	3 699	1 735	8 247
40	124 230	2 090 153	664 510	5 668	3 484	1 452	8 247
41	118 920	1 965 923	540 280	5 426	3 277	1 180	8 242
42	113 550	1 847 003	421 360	5 181	3 078	920	8 234
43	108 110	1 733 453	307 810	4 933	2 889	672	8 223
44	102 620	1 625 343	199 700	4 683	2 709	436	8 209
45	97 080	1 522 723	97 080	4 430	2 538	212	8 190
46	91 480	1 425 643		4 174	2 376		8 165
47	85 830	1 334 163		3 916	2 224		8 136
48	80 130	1 248 333		3 656	2 081		8 099
49	74 400	1 168 203		3 394	1 947		8 055
50	68 640	1 093 803		3 130	1 823		8 001

The results present some curious contrasts between the life-curves for the total population in the loyal States and in the insurgent States, which may be best recognized by reference to the appended chart, marked *E*. This chart exhibits the number of white males at each year of age from 18 to 50, corresponding to each 10 000 at the age of 18. It will be seen at once that the curvature of the line representing the population of the insurgent States is in the direction opposite to that of the lines belonging to the loyal States and to the whole country. The dotted line is straight, and shows what the distribution would be, did it follow a regular arithmetical progression. To what extent this difference may be due to immigration from Europe, which has been chiefly to the Free States, I will not undertake to estimate. We have seen, however, that the law of distribution of our volunteer troops according to ages was essentially the same for those States to which immigration is greatest as for those to which it is least.

The construction of all the curves laid down on the accompanying charts will be manifest without explanation. For those ordinates which belong to the respective ages they give the corresponding numbers.

APPENDIX.

ON THE AGES OF A POPULATION.

In the course of the preceding investigation, the interesting question as to the general distribution of a population by ages became prominent; and the inquiry continually suggested itself, how far any simple formula might be capable of representing the observed numbers for all ages of life. This has incidentally led to the detection of what seems to be the true law, which, although not strictly pertaining to the subject in hand, seems yet to possess sufficient practical value and importance in its indirect bearing to justify its introduction here, — the more especially, since endeavors to obtain information on this point elsewhere have proved fruitless.

It appears that, in a population at all homogeneous in its character, the number of persons under the age n years may be represented by the simple expression

$$N = a \sin n k^n \theta$$

in which a denotes the total number of the population, while k and θ are constants peculiar to the country and epoch. The quantity θ is an angle somewhat larger than 1°, and k is a number, generally a little less than unity.

For the special case $k = 1$, the formula becomes

$$N = a \sin n \theta$$

containing only one unknown quantity, the angle θ, to be determined by investigation.

A very peculiar characteristic of this law is recognizable in the circumstance that the number at any given age appears to be strictly proportional to the whole population; so that the expectation of life, for the average individual, is as well represented as is the general distribution by ages of the total number of individuals, of whom the population is composed.

Investigating the values of the constants k and θ for the people of the United States at each of the last four enumerations, we find

Date	k	θ
1830	0.9918	2°.0524
1840	0.9921	1°.9747
1850	0.9932	1°.8361
1860	0.9941	1°.7307.

The census of 1820 is not sufficiently distinct, in the assortment by ages, to permit a determination of the constants, but the indications are clear that a proper enumeration would have afforded results in conformity with the preceding series; the value of k being smaller, and that of θ larger than for the population in 1830.

The curious fact thus becomes evident, that our population has, during the last forty years or more, been gradually assimilating itself to the normal type represented by $k = 1$; growing, or developing itself, so to speak, toward a compliance with the simple law

$$N = a \sin n\theta$$

in which the value of θ indicates the longevity of the people, since, according to the formula, the entire population becomes extinct at the age when $n\theta = 90°$.

How far immigration has affected the values of the constants for the United States we will not now inquire. Were the tendency to immigrate independent of age, no appreciable influence could be traced to this source; and the character of the immigration into this country seems to have been such as to exhibit no great preponderance or deficiency for any one period of life, except perhaps that there is some deficiency in the relative number at the most advanced ages. But the accessions to our population from Ireland and Germany appear to have been in most cases by families, and not composed chiefly of persons in the prime of life or fullness of strength, as is the case in very new countries.

The English people appearing to afford a fair specimen of a permanent and normal population, the last two censuses of England and Wales were examined, and with the following result: —

Date	k	θ
1851	0.9957	1°.4702
1861	0.9962	1°.4316.

Thus a similar phenomenon is manifested by the English enumerations to that exhibited by the American census-returns; the values of k approaching unity, and those of θ diminishing. The smaller value of the angle θ

indicates a longer duration of life in that country; but k, the modulus of the change by geometric progression, was not larger for England in 1851 than it bids fair to be for the United States in ten years from the present time.

Passing next to the French population, we find the value $k = 1$ as the result alike of the last three enumerations, the values of θ being

in 1851	1°.0553
" 1856	1°.0556
" 1861	1°.0473.

The remarkable peculiarity of the life-curve for France, as regards the small infantile mortality, is well exhibited by the chart F, which shows the number living, at each year of age, for every million in the population. The several curves of this chart represent the distribution of ages for the United States in 1830 and 1860, (those for the intermediate decades being omitted to avoid confusion,) for England in 1861, and for France. The English curve for 1851 would differ too slightly from that for 1861 to be conveniently distinguished on the chart; and the French curves for 1851, 1856, and 1861 would be undistinguishable from one another.

The chart G shows the corresponding values of N, (the number under each year of age,) for each nation, and clearly manifests the differences in the law, corresponding to the diversity in the constants.

The tables of population deduced from the census-returns already cited, together with the values given by the formula, are here appended, reduced however, in each case, to the scale of one million of population. The differences are given in decimals of the census-numbers, and the accordance between the formula and the recorded numbers will be manifest at the first inspection. The chief discrepancies will be found in the French tables, for the ages

exceeding 50,	in the census of 1851
" 55,	" " " " 1856
" 60,	" " " " 1861.

This curious circumstance and the nature of the discordances suggest some historical explanation; which the disturbed condition of the French nation at the period corresponding to the birth of this portion of the population seems to render plausible.

TABLE XXII.

Ages of the Population of the United States, as deduced from the Census Returns of 1830 *and* 1840.

AGE.	Census of 1830.			Census of 1840.		
	Proportional numbers.		Difference. (C. — O.)	Proportional numbers.		Difference. (C. — O.)
	Observed.	Calculated.		Observed.	Calculated.	
0 – 5	17977	17082	−0.050	17437	16334	−0.063
5 – 10	14576	15254	+0.046	14173	14651	+0.034
10 – 15	12452	13280	+0.014	12094	12931	+0.069
15 – 20	11147	11318	+0.024	10911	11205	+0.027
20 – 30	17752	17244	−0.029	18155	17456	−0.038
30 – 40	10908	11287	+0.035	11597	11790	+0.017
40 – 50	6886	6932	+0.007	7320	7466	+0.020
50 – 60	4308	3973	−0.078	4365	4389	+0.005
60 – 70	2525	2100	−0.168	2449	2343	−0.043
70 – 80	1104	992	−0.100	1132	1067	−0.058
80 & over	365	540		367	368	

TABLE XXIII.

Ages of the Population of the United States, as deduced from the Census Returns of 1850 *and* 1860.

AGE.	Census of 1850.			Census of 1860.		
	Proportional numbers.		Difference. (C. — O.)	Proportional numbers.		Difference. (C. — O.)
	Observed.	Calculated.		Observed.	Calculated.	
0 – 1	2751	3170	+0.152	2998	3003	+0.002
1 – 5	12070	12215	+0.012	12300	11608	−0.056
5 – 10	13836	14102	+0.019	13117	13484	+0.028
10 – 15	12292	12564	+0.022	11588	12206	+0.053
15 – 20	10892	10990	+0.009	10625	10853	+0.021
20 – 30	18562	17505	−0.057	18242	17692	−0.030
30 – 40	12368	12225	−0.012	13012	12760	−0.019
40 – 50	8130	8019	−0.013	8496	8618	+0.014
50 – 60	4903	4883	−0.041	5214	5366	+0.029
60 – 70	2667	2695	+0.010	2910	2953	+0.015
70 – 80	1147	1250	+0.090	1158	1261	+0.089
80 & over	382	382		340	196	

TABLE XXIV.

Ages of the Population of England and Wales,

as deduced from the Census Returns of 1851 *and* 1861.

AGE.	Census of 1851.			Census of 1861.		
	Proportional numbers.		Difference. (C. — O.)	Proportional numbers.		Difference. (C. — O.)
	Observed	Calculated.		Observed.	Calculated.	
0 — 5	13006	12533	−0.035	13352	12245	−0.083
5 — 10	11590	11800	+0.018	11588	11575	−0.001
10 — 15	10616	10987	+0.034	10415	10819	+0.040
15 — 20	9832	10079	+0.013	9688	10007	+0.032
20 — 25	9441	9114	−0.036	9317	9108	−0.023
25 — 30	8307	8170	−0.017	7932	8178	+0.030
30 — 35	7168	7179	+0.004	6950	7282	+0.046
35 — 40	6083	7273	+0.030	6111	6352	+0.038
40 — 45	5393	5378	−0.003	5638	5506	−0.024
45 — 50	4440	4546	+0.023	4617	4705	+0.019
50 — 55	3934	3782	−0.040	3995	3820	−0.046
55 — 60	2919	3061	+0.046	3039	3245	+0.063
60 — 65	2668	2426	−0.100	2751	2512	−0.095
65 — 70	1815	1841	+0.014	1862	1910	+0.025
70 — 75	1386	1332	−0.041	1391	1355	−0.026
75 — 80	809	876	+0.076	794	869	+0.086
80 — 85	410	481	+0.173	394	435	+0.104
85 & over.	183	142		146	77	
Total,	100000	100000		100000	100000	

FORMULAS

For 1851, $N = 100\,000 \sin n\ (0.99575)^n \,.\, 1°.4702.$

1861, $N = 100\,000 \sin n\ (0.99616)^n \,.\, 1°.4316.$

TABLE XXV.

Ages of the Population of France,

as deduced from the Census Returns of 1851, 1856, *and* 1861.

AGE.	Census of 1851.			Census of 1856.			Census of 1861.		
	Proportional numbers.		Difference. (C. — O.)	Proportional numbers.		Difference. (C. — O.)	Proportional numbers.		Difference. (C. — O.)
	Observed.	Calculated.		Observed.	Calculated.		Observed.	Calculated.	
0 – 5	9291	9208	−0.009	9568	9200	−0.038	9677	9124	−0.088
5 – 10	9216	9130	−0.009	9120	9119	0.000	8767	9052	+0.032
10 – 15	8800	8946	+0.016	8821	8965	+0.016	8668	8898	+0.027
15 – 20	8805	8716	−0.010	8530	8736	+0.024	8701	8722	+0.002
20 – 25	8326	8437	+0.013	8077	8427	+0.043	8237	8374	+0.017
25 – 30	8020	8036	+0.002	8075	8051	−0.003	7857	8005	+0.019
30 – 35	7565	7616	+0.007	7575	7614	+0.005	7421	7564	+0.019
35 – 40	7188	7105	−0.012	7255	7092	−0.022	7098	7071	−0.004
40 – 45	6596	6534	−0.009	6656	6526	−0.020	6625	6514	−0.017
45 – 50	5869	5890	+0.004	6041	5902	−0.023	6155	5900	−0.041
50 – 55	5782	5233	−0.095	5317	5228	−0.017	5382	5254	−0.024
55 – 60	4390	4512	+0.028	4838	4503	−0.069	4559	4518	−0.009
60 – 65	3670	3753	+0.023	3734	3753	+0.005	4160	3790	−0.090
65 – 70	2785	2954	+0.059	2757	2958	+0.076	2941	3016	+0.025
70 – 75	1952	2148	+0.100	1902	2145	+0.128	1940	2218	+0.143
75 – 80	1062	1313	+0.239	1088	1312	+0.205	1123	1398	+0.245
80 – 85	480	468	−0.025	453	468	−0.033	490	568	+0.159
85 & over.	203	1		193	1		199	14	
Total,	100000	100000		100000	100000		100000	100000	

FORMULAS

For 1851, $N = 100\,000 \sin n\,(1°.0553)$.

1856, $N = 100\,000 \sin n\,(1°.0556)$.

1861, $N = 100\,000 \sin n\,(1°.0473)$.

The agreement of the observed numbers with those given by our formula is indicated by the quantities in the columns headed $C. - O.$ (i. e. Computed *minus* Observed), and appears to be entirely within the limits of probable error in the enumeration, — if we except those discordances for the French census already alluded to. It affords a strong argument for belief that the true form of the normal life-curve is closely represented by the sine-formula.

The only other statistics of ages for European populations, which have been conveniently accessible, are contained in the abstract of the Prussian census of 1852, given by Brachelli, in the second volume of his *Deutsche Staatenkunde*. A discussion of the numbers there recorded gives

$$k = 0.9960, \qquad \theta = 1°.4702,$$

these values being closely similar to those for England and Wales in 1851.

It is manifest that if the number under any given age n be represented by the expression

$$N = a \sin n k^n \theta$$

the number between the ages n and $n+1$ will be expressed by

$$2a \sin \tfrac{1}{2} k^n (k n + k - n)\theta . \cos \tfrac{1}{2} k^n (k n + k + n)\theta,$$

and the mortality at the same period, by the finite difference of this quantity.

But when θ becomes unity, these values are greatly simplified, and we have

Population under the age n years	$=$	$a \sin n\theta$
Population at " " " "	$=$	$2a \sin \tfrac{1}{2}\theta \cos (n + \tfrac{1}{2})\theta$
Mortality " " " " "	$=$	$4a \sin (n+1)\theta \sin^2 \tfrac{1}{2}\theta.$

According to the formula here presented, the life-curve for advanced ages bears no similarity to an asymptote, but ceases abruptly when the quantity $n k^n \theta = 90°$; or for the case of $k = 1$, when $n = \frac{90°}{\theta}$. This indicates that all ages above this limit are exceptional, and to be regarded in the same light as deviations from the theoretical number at other periods of life.

The many paths of research offered by the residual discordances from the formula must be passed by on this occasion, with the single remark that they offer indications of abundant reward for any explorer.

It is proper to add, that in these investigations, as in those on the Ages of Volunteers, the computations have been almost exclusively carried on by Mr. J. N. Stockwell.

CAMBRIDGE, January, 1866.

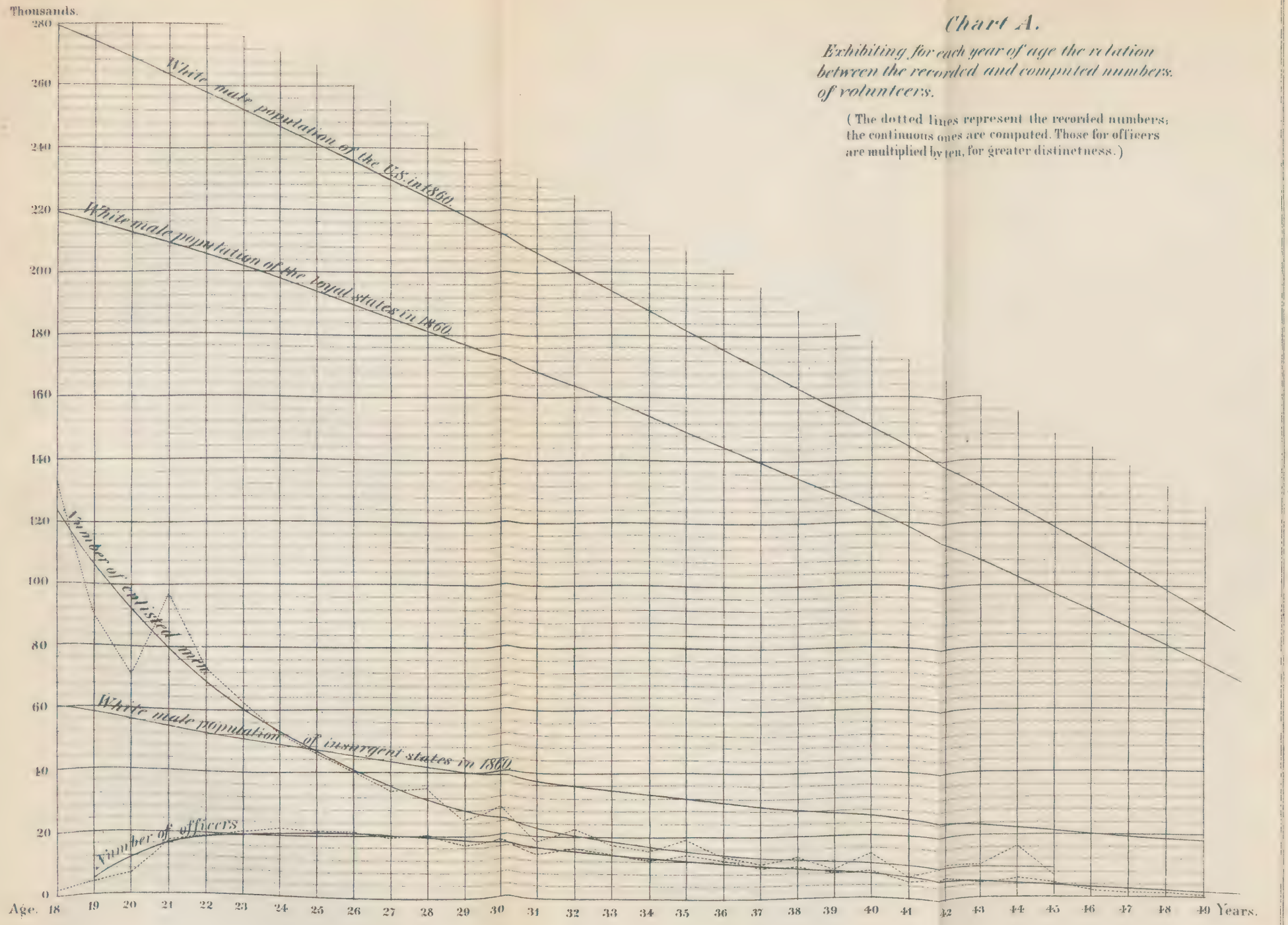


Chart A.

Exhibiting for each year of age the relation between the recorded and computed numbers of volunteers.

(The dotted lines represent the recorded numbers; the continuous ones are computed. Those for officers are multiplied by ten, for greater distinctness.)

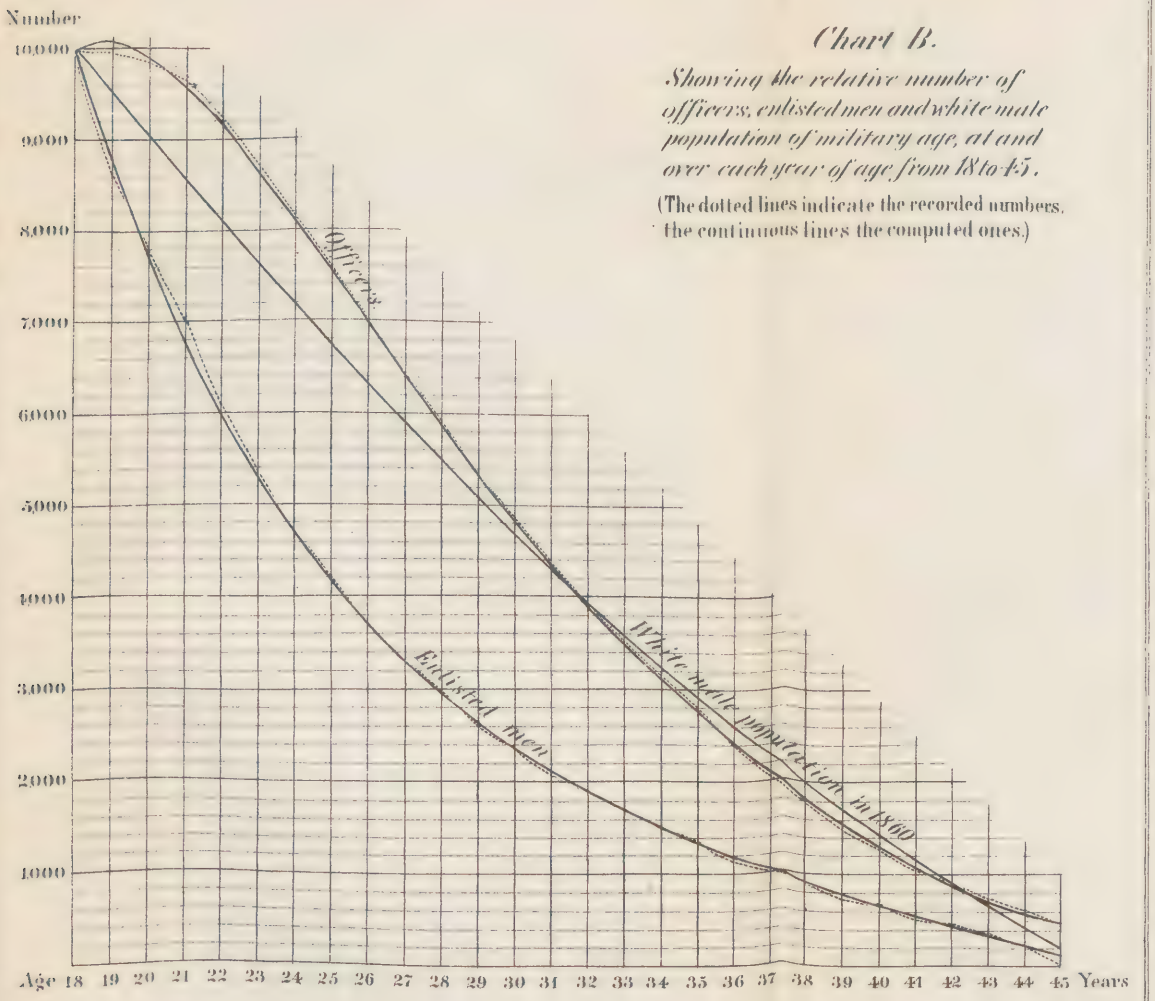


Chart B.

Showing the relative number of officers, enlisted men and white male population of military age, at and over each year of age from 18 to 45.

(The dotted lines indicate the recorded numbers, the continuous lines the computed ones.)

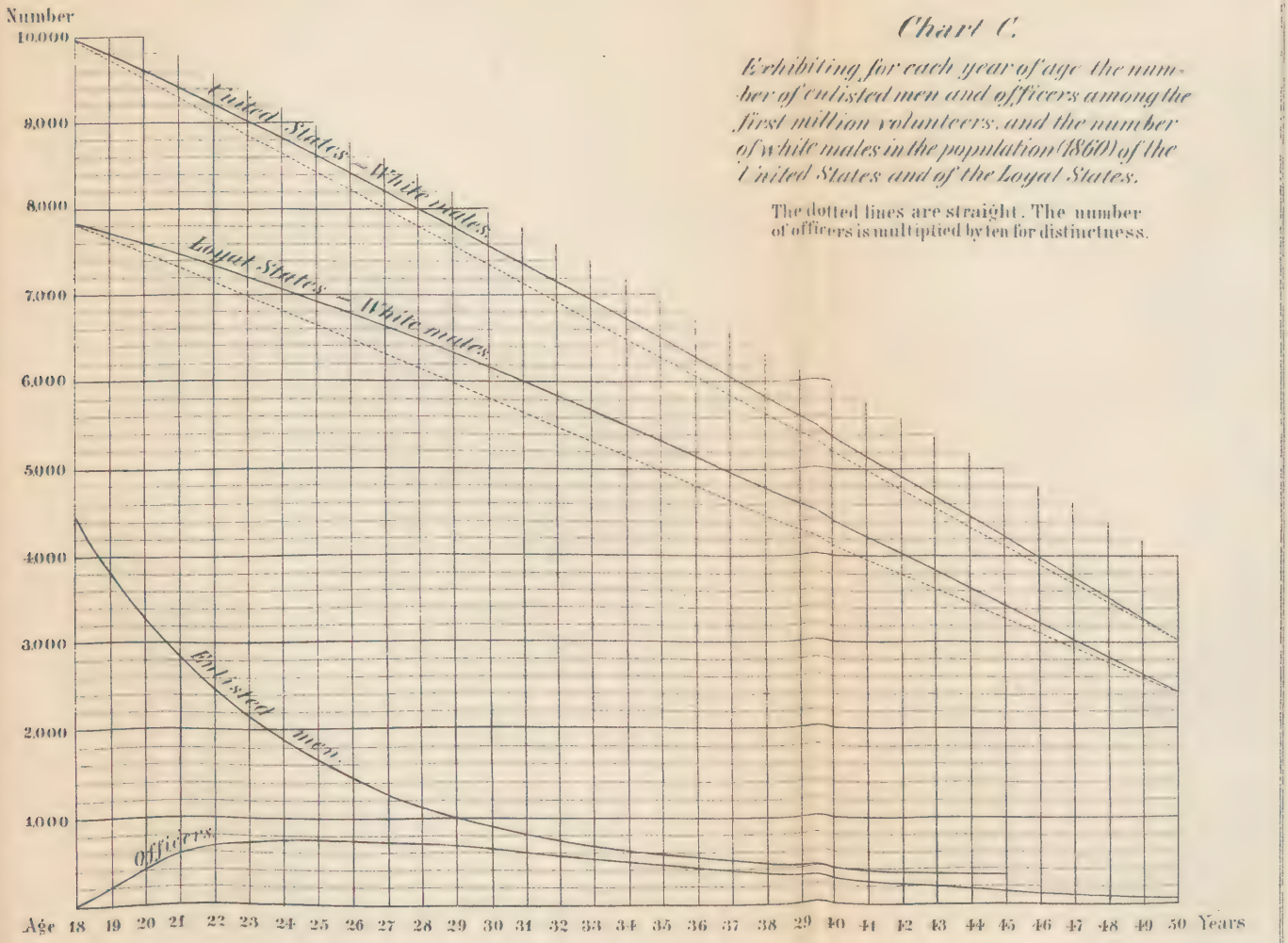


Chart C.

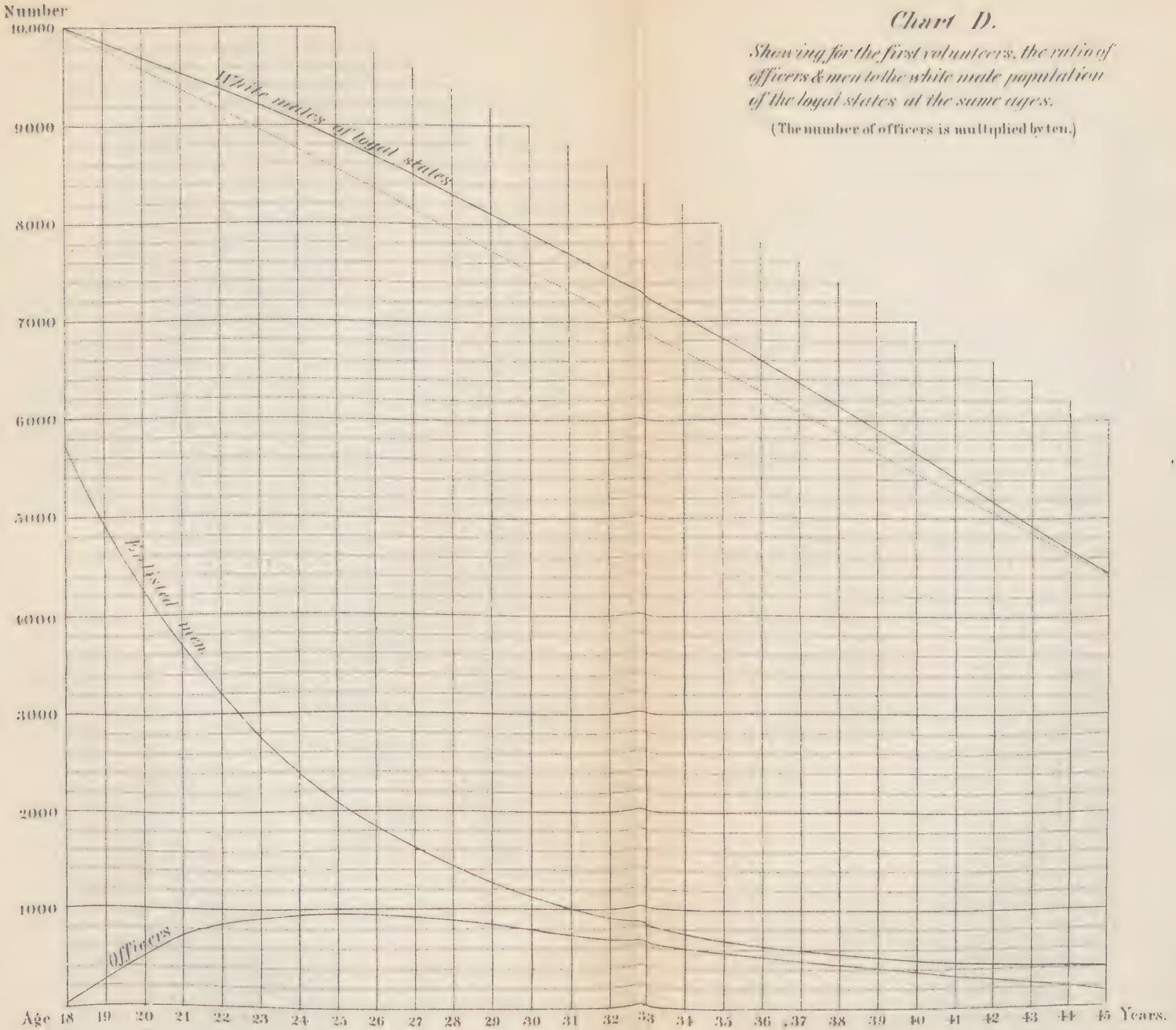
Exhibiting for each year of age the number of enlisted men and officers among the first million volunteers, and the number of white males in the population (1860) of the United States and of the Loyal States.

The dotted lines are straight. The number of officers is multiplied by ten for distinctness.

Chart D.

Showing for the first volunteers, the ratio of officers & men to the white male population of the loyal states at the same ages.

(The number of officers is multiplied by ten.)



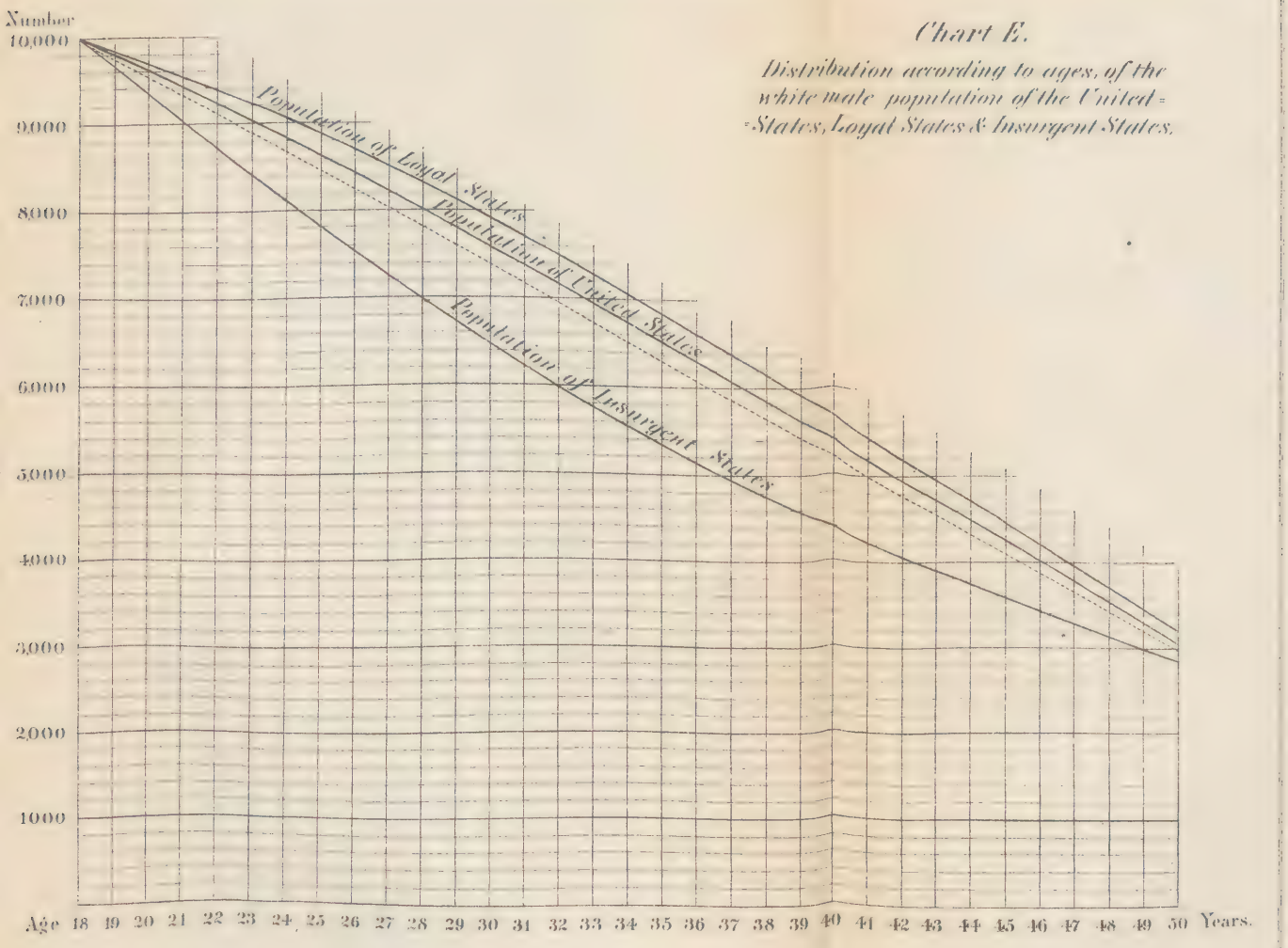


Chart E.

Distribution according to ages, of the white male population of the United States, Loyal States & Insurgent States.

Chart F.

Showing the distribution of a million of population, by years of age, for the United States, England, & France.

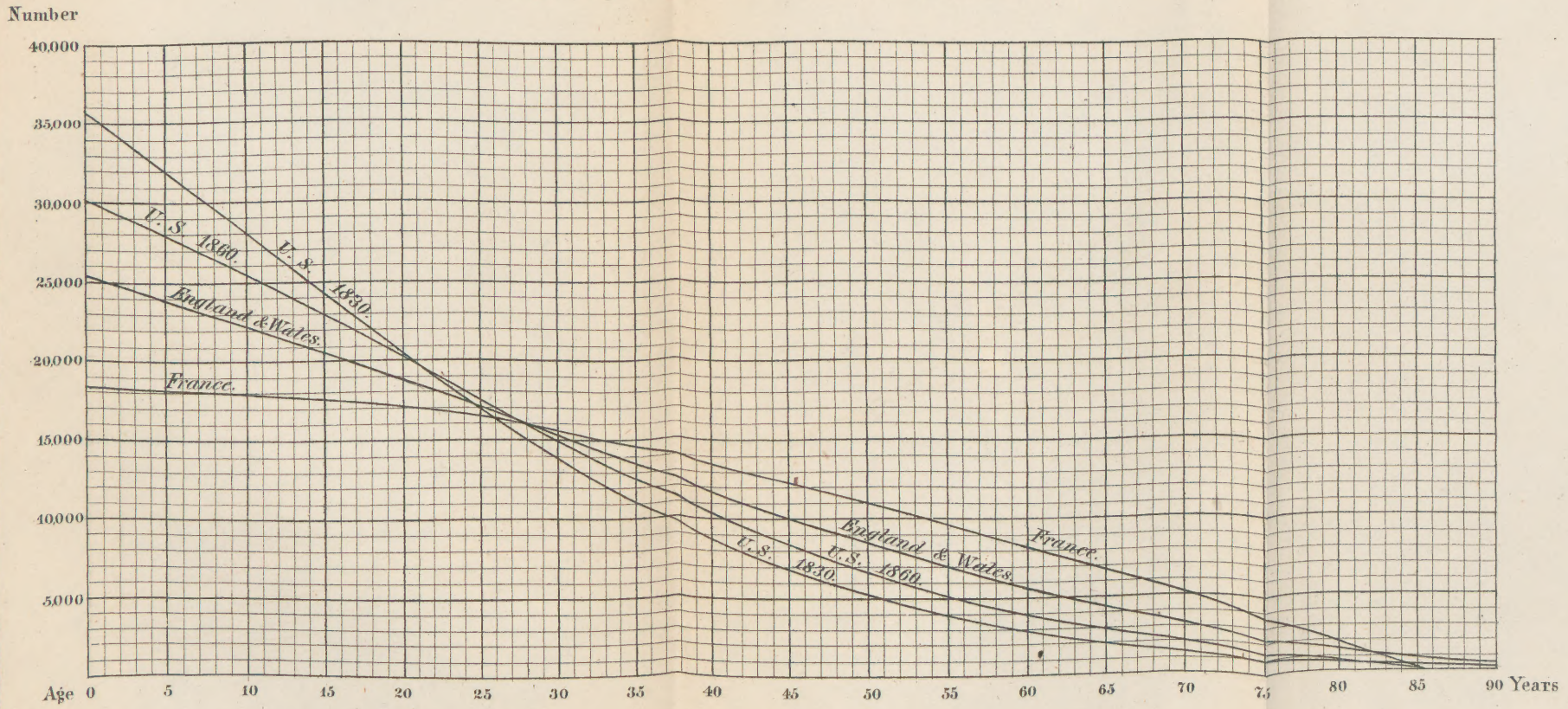


Chart G.

Exhibiting the relative number of the population under each successive year of age, for the United States, England, & France.